MAT 137Y: Calculus!
Problem Set 8
Due on Thursday, February 28 by 11:59pm via crowdmark

Instructions:

- You will need to submit your solutions electronically. For instructions, see `http://uoft.me/CM137` . Make sure you understand how to submit and that you try the system ahead of time. If you leave it for the last minute and you run into technical problems, you will be late. There are no extensions for any reason.
- You will need to submit your answer to each question separately.
- This problem set is about volumes and sequences (Playlists 10 and 11)

1. Let R be the bounded region enclosed by the curves $y = x$ and $2y^2 = 3x + 2$. We rotate R around the y-axis. Compute the volume of the resulting solid.

   Notes: Careful! The region R intersects three quadrants. The axis of rotation cuts across the region, and you need to figure out what that does to the solid.

   The difficulty of this problem is the set up. The volume can be written as an integral (or a sum of integrals) of polynomials. Make sure you explain all the process to get it to that form. If you get it to that form, you do not need to perform the integration in detail. You may jump directly from that expression to the final answer.

2. Write a proof for the following theorem

   Theorem Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.
   - IF $\{a_n\}_{n=0}^{\infty}$ is eventually decreasing and not bounded below,
   - THEN $\{a_n\}_{n=0}^{\infty}$ is divergent to $-\infty$.

   Write a proof directly from the definitions. You will need to use the definitions of "eventually decreasing", "not bounded below", and "divergent to $-\infty$". As usual, pay attention to the proof structure.

3. In this problem we only consider sequences that are **positive and divergent to** ∞.

   For each of the following statements, decide whether they are true or false. If true, prove it. If false, give a counterexample.

(a) IF $\{x_n\}_n$, $\{y_n\}_n$, $\{z_n\}_n$ are sequences such that $x_n << y_n$ and $y_n << z_n$ THEN $x_n << z_n$.

(b) For every sequence $\{x_n\}_n$, there exists a sequence $\{y_n\}_n$ such that $y_n << x_n$

(c) IF $\{x_n\}_n$ and $\{y_n\}_n$ are sequences such that $x_n << y_n$ THEN there exists a sequence $\{z_n\}_n$ such that $x_n << z_n << y_n$.

(d) For every sequence $\{x_n\}_n$, there exists a sequence $\{y_n\}_n$ such that for every $a > 0$, $(x_n)^a << y_n$

Reminder: In this problem we are only considering sequences that are **positive and divergent to** ∞.