

MAT 137Y: Calculus!
Problem Set 6 - Common errors

[Q1] The most common error was simply poor writing. Many students had the right idea but the writing was careless or incorrect. By now you should have learned the importance of writing carefully and of proof structure.

In addition, do not assume the limit exists. We even warned you in the question not to do that.

[Q2a] Use the definition of upper integral. The upper is **not** one of the upper sums.

As always, you need to justify your answer.

[Q2b] By now you should have learned the importance of proof structure. You should know that variables need to be introduced in order. You should be clear on which variables are arbitrary and fixed, and which variables depend on what.

Other things to notice

- The set B does not have a minimum or “first” element.
- If your partition includes, for example $P = \left\{ \dots, \frac{1}{k} + \varepsilon, \frac{1}{k-1} - \varepsilon, \dots \right\}$, you need to make sure that $\frac{1}{k} + \varepsilon < \frac{1}{k-1} - \varepsilon$. Otherwise it is not a well-defined partition.
- The partition will depend on a fixed value of n and a fixed value of ε .

[Q2c] – Use the result you have already proven in Q2b. If you start from scratch again, it shows poor understanding of what a proof is.

- Once again, you need to pay attention to quantifiers: what are you fixing and in which order? What are you choosing? What depends on what?

You cannot simply write something like $\varepsilon = \frac{1}{n} + \varepsilon'$. Instead, be much more careful: first fix ε , then say how you choose n and ε' depending on ε .

- Any answer with limits is probably wrong. You cannot take $\lim_{n \rightarrow \infty}$ in Q2b, because the partition on Q2b depends on n .

[Q2d] – Your argument will probably have to go in two parts. From your answer to Q2c alone, you can only conclude that the lower integral is ≥ 1 . Why is it exactly 1?

- Notice that the lower integral is defined as the supremum of **all** the lower sums. In Q2b and Q2c you have not used all the lower sums.