MAT 137Y: Calculus!
Problem Set 5 - Common errors

[Q1] There were three very common errors:

- We need to optimize the quotient of distances (see sample solution), but we need to explain why. Some of you argued that we want S_I to be large and S_A to be small, so we are going to look for the maximum of the quotient $\frac{S_I}{S_A}$. That is not a complete explanation: using the same argument we could have decided to optimize the difference $S_I - S_A$ instead, which gives a different answer. Why choose the quotient specifically?
- Once you set the quantity to optimize, and you find that it has one single critical point in the domain, you are not done. How do you know that the maximum/minimum of the function happens at that critical point? You still need to check the endpoints and invoke the EVT, or check the sign of the derivative to the left and right of the critical point, or check the second derivative, or ...
- If your answer depends on Ivan's velocity, your full set up is wrong and you get no credit. Alfonso does not know Ivan's velocity, so such an answer is useless for him to decide where to try to exit the pool.

[Q2a] The error is that we cannot use L'Hôpital's Rule because we do not know whether $\lim_{x \to a} h'(x)$ exists. To get credit, you need to say that we are not allowed to use L'Hôpital's Rule and why.

Only saying that "we do not know whether $\lim_{x \to a} h'(x)$ exists" is not enough.

[Q2b] The statement

- "Let I be an interval. Let h be a differentiable function on I.
  If $\lim_{x \to a} h'(x)$ exists, then h' is continuous"

is meaningless. Are you talking about the limit for one specific a or for all a? The statement

- "Let I be an interval. Let h be a differentiable function on I. Let $a \in I$.
  If $\lim_{x \to a} h'(x)$ exists, then h' is continuous"

is incorrect. If you fix a and ask for $\lim_{x \to a} h'(x)$ to exist at that specific point, then you can only conclude that h' is continuous at a.

[Q3b] You need to justify why your counterexample works. Specifically, you need to prove that your counterexample satisfies that $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \pm\infty$ but $\lim_{x\to a} f'(x) \neq \pm\infty$.

Notice that if you have a sum of functions and one of them does not have a limit, this is not enough to conclude anything about the limit of the sum.

"Proof by graph" is not enough. For complicated functions like this one, we cannot predict the behaviour of the derivative based on the graph. There are functions with very similar-looking graph but with very different derivatives.

[Q4a] You need to justify why your counterexample works. Specifically, you need to prove that your counterexample satisfies that $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} f'(x) \neq 0$.

"Proof by graph" is not enough. For complicated functions like this one, we cannot predict the behaviour of the derivative based on the graph. For example, $f(x) = \frac{\sin(x^2)}{x}$ and $g(x) = \frac{\sin(x^{1.5})}{x}$ have very similar-looking graphs, but $\lim_{x\to\infty} f'(x)$ DNE while $\lim_{x\to\infty} g'(x) = 0$.

[Q4b] You need to justify why your counterexample works.