MAT 137Y: Calculus!
Problem Set 2 - Common errors

[Q0] About 4% of you did not get credit for this question in PS1 or PS2! The question asked to copy the statement and sign it. We were generous in what counts as a signature, but if you did not sign it, then you get no point. Please get in the habit of reading instructions.

[Q2a] You are trying to prove that $\lim_{x\to a} g(x)$ exists. You may not assume it exist! For example, if you write, at any time that

$$\lim_{x\to a} [f(x)+g(x)] = \left[\lim_{x\to a} f(x)\right] + \left[\lim_{x\to a} g(x)\right]$$

then your proof is wrong. You are assuming the conclusion.

Instead you may write

$$\lim_{x\to a} g(x) = \left[\lim_{x\to a} (f(x)+g(x))\right] - \left[\lim_{x\to a} f(x)\right]$$

because, to do so, the only things you are assuming are that $\lim_{x\to a} [f(x)+g(x)]$ and $\lim_{x\to a} f(x)$ exist, and those are things you may assume.

[Q2b]

- The statement is false. It is not enough to notice that the proof from Q2a does not work here. Perhaps there is a different reason why the statement is true. In order to prove the statement is false you need to provide one specific counterexample.
- In addition, some of you used the argument that if $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ DNE. This is not true without extra conditions.

[Q3]

- Your rough work is not part of your proof. Your rough work is what you do on a separate piece of paper to figure out how to write your proof. We will grade only your actual proof, and it must be self-contained.
- Your proof must have the correct structure. First, fix an arbitrary ε, then say what δ is, then verify it works.
- **Extremely common error:** In general, given a polynomial P it is NOT true that $a < x < b \implies P(a) < P(x) < P(b)$. This is true for *increasing* polynomials, but not for arbitrary ones.

[Q4]

- Think about proof structure. You are trying to prove that

$$\forall L \in \mathbb{R},\ \exists \varepsilon > 0,\ \text{s.t. } \forall \delta > 0, \exists x \in \mathbb{R} \text{ s.t. } \boxed{\text{2 inequalities}}$$

  In your proof:

  1. Fix an arbitrary L
  2. Say what ε you take
  3. Fix an arbitrary δ
  4. Say what x you take
  5. Verify it works

- The order matters. A variable may only depend on the variables that have already been fixed before it.
- Remember that quantified variables do not have any intrinsic meaning. In particular, "the delta in the definition of $\lim_{x \to a} f(x) = \infty$" and "the delta in the definition of $\lim_{x \to a}$ DNE" are not the same.
- Finally, consider the definition of $\lim_{x \to a} f(x) = \infty$:

$$\forall M \in \mathbb{R},\ \exists \delta > 0,\ \text{s.t. } 0 < |x - a| < \delta \implies f(x) > M$$

  If you were trying to *prove* that $\lim_{x \to a} f(x) = \infty$, you would begin your proof by fixing an arbitrary $M \in \mathbb{R}$. But here you are not trying to prove it: you already know it is true. Setting M to be arbitrary is unnecessary and useless. Instead, you get to choose M to be whatever you want, and then you get a value of δ for free.