MAT 137Y: Calculus!
Problem Set 1 - Some comments after grading

Some of you have probably received a grade much lower than expected. Do not despair yet!

This is normal. You are learning to write proofs for the first time. This is difficult. It takes time to master. If you feel that you are making progress, if things are starting to make sense, if you read the sample solutions and comments and you understand where you went wrong, don't be disheartened.

- General comment: Make sure your files are legible and properly rotated. Some of your submissions were of such poor image quality that we could not read them.

[Q0] The question asked to copy the statement and sign it. We were generous in what counts as a signature, but if you did not sign it, then you get no point. Please get n the habit of reading instructons.

[Q2] There was one common misunderstandings of the definitions in this problem.

Some of you tried to argue that the definition of "f is happier than g" is always vacuously true. Be careful. The definition says

$$\text{“}\exists x \in \mathbb{R} \text{ s.t. } \forall y \in \mathbb{R}, \qquad [x < y \implies f(y) > g(y)]\text{”}$$

It does not say

$$\text{“}\exists x \in \mathbb{R} \text{ s.t. } \forall y \in \mathbb{R}, \qquad x < y\text{”}$$

or even

$$\text{“}[\exists x \in \mathbb{R} \text{ s.t. } \forall y \in \mathbb{R}, x < y] \qquad \implies f(y) > g(y)\text{”}$$

The last one is not even a meaningful statement.

Various of you correctly noticed that there isn't a value of x which satisfies

$$\text{“}\forall y \in \mathbb{R}, x < y\text{”}$$

That is irrelevant. You cannot separate pieces of a statement.

If you are having trouble with this, consider the following two statements:

1. The following is true: "$\forall t \in \mathbb{Z},\ t > 5 \implies t > 2$"
2. The following is false: "$\forall t \in \mathbb{Z},\ t > 5$"

[Q2] There was another common misunderstanding in the notation in this problem.

The variables 'x' and 'y' appear in the middle of a long statement and they are quantified. They do not have any intrinsic meaning. We are never saying, for example, '$f(y) > g(y)$' by itself. And, very importantly, "the y in the definition of happier" is not the same as "the y in the definition of luckier".

If you are having trouble with this, consider the following two statements:

1. "$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ s.t. $x < y$"
2. "$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ s.t. $x > y$"

Both of the are true! Hopefully this convinces you that 'y' does not have a separate meaning by itself.

[Q2a] To prove the statement is false, you need to provide one counterexample. Give ONE single pair of functions f and g, and then verify two thing separately: that f is luckier than g, and that f is not happier than g. That's all.

You may not use one pair of functions to prove that f is luckier than g, and a different pair to prove that g is luckier than f.

[Q2b] Many of you had a correct idea, but the structure of the proof was particularly bad for this question.

- You want to prove that f is luckier than g. Your proof **MUST** begin by fixing an arbitrary $x \in \mathbb{R}$. Then you have to say what y is in terms of x (or explain why there exists a y with the desired properties.)
- Explain your notation. Introduce your variables. Explain where the things you write come from.
- Most importantly, 'the x' in the definition of happier is not the same as 'the x' in the definition of luckier. So use different letters. And explain your notation.

[Q2c] The proof structure for this question was better, but some of you also had trouble.

You want to prove that "For every function f, there exists a function g such that, for every $t \in \mathbb{R}$... "

- Your proof **MUST** begin by fixing an arbitrary function f.
- Then you need to say what g is. Notice that g may depend on f, but g may not depend on t.
- Then you fix an arbitrary value of $t \in \mathbb{R}$.

Next, you need to prove that g is happier than f_t. For that you need to say what x is (which may depend on t) and verify it works.

[Q3] Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing. Explain what you are doing.

The induction step works on one value of n at a time. In you induction step, **YOU FIX ONE POSITIVE INTEGER** n, you assume that $2^{3n} + 6$ is a multiple of 7, and you prove that $2^{3(n+1)} + 6$ is also a multiple of 7 for that specific value of n. If, instead, you write

"Assume $\forall n$, $2^{3n} + 6$ is a multiple of 7",

then your proof is wrong. You are assuming too much (you are assuming what you want to prove!)

Also, explain what you are doing.