1. Let \( \{a_n\}_{n=1}^{\infty} \) be a sequence. Write down the formal definition of the following concepts. You have already seen some of these in lecture.

(a) The sequence is convergent.
(b) The sequence is divergent.
(c) The sequence is divergent to \( \infty \).
(d) The sequence is divergent to \( -\infty \).
(e) The sequence is increasing.
(f) The sequence is decreasing.
(g) The sequence is non-decreasing.
(h) The sequence isn’t decreasing.
(i) The sequence is bounded above.
(j) The sequence is not bounded above.
(k) The sequence is bounded.

Note: Questions 1e, 1g and 1h are all different.

2. Prove formally the following statements. On each proof, you need to assume the hypotheses (use the definition!) and show the conclusions are true (use the definitions!). You may have already seen some of these in lecture.

(a) If a sequence is increasing and bounded above, then it is convergent.
(b) If a sequence is increasing and not bounded above, then it is divergent to infinity.
(c) If a sequence is convergent, then it is bounded.