1. Negate the following statement without using any negative words ("no", "not", "none", etc.):
"I have a friend all of whose former boyfriends had at least two siblings with exactly three different vowels in their name."

2. Let $A$ be a subset of the real numbers. Iva, Peter, Trefor, and Fedya tell us when they like it:

- Iva likes $A$ if and only if $\exists a \in \mathbb{R}$ such that $\forall x \in A, a \leq x$.
- Peter likes $A$ if and only if $\exists a \in A$ such that $\forall x \in A, a \leq x$.
- Trefor likes $A$ if and only if $\exists a \in \mathbb{R}$ such that $\forall x \in A, a < x$.
- Fedya likes $A$ if and only if $\exists a \in A$ such that $\forall x \in A, a < x$.

Answer the following questions about Iva, Peter, Trefor, and Fedya. Prove your answers.

(a) Is there any of them who likes every subset of the real numbers?
(b) Is there any of them who does not like any subset of the real numbers?
(c) Who likes the empty set?
(d) Are there two different people who like exactly the same subsets of the real numbers?
(e) Is the following statement true?
   For every \( \mathcal{A} \subseteq \mathbb{R} \), if Iva likes \( \mathcal{A} \) then Peter likes \( \mathcal{A} \).
(f) Is the following statement true?
   For every \( \mathcal{A} \subseteq \mathbb{R} \), if Peter likes \( \mathcal{A} \) then Iva likes \( \mathcal{A} \).

Suggestion: Before you do anything else, try to understand with words which types of subsets each of the four people like.

3. The following proof is wrong. Explain why it is wrong. Then rewrite the proof so that it is correct.

**Theorem:** If \( \mathcal{A}_1 \) and \( \mathcal{A}_2 \) are two sets of real numbers that Iva likes, then Iva also likes \( \mathcal{A}_1 \cup \mathcal{A}_2 \).

**Proof:**

- Since Iva likes \( \mathcal{A}_1 \), we know that for all \( x \in \mathcal{A}_1 \), \( a \leq x \). (I)
- Since Iva likes \( \mathcal{A}_2 \), we know that for all \( x \in \mathcal{A}_2 \), \( a \leq x \). (II)
- We need to prove that for all \( x \in \mathcal{A}_1 \cup \mathcal{A}_2 \), \( a \leq x \).
  
  Let \( x \in \mathcal{A}_1 \cup \mathcal{A}_2 \).
  
  - Either \( x \in \mathcal{A}_1 \) — in this case \( a \leq x \) by (I).
  - Or \( x \in \mathcal{A}_2 \) — in this case \( a \leq x \) by (II).

  In both cases \( a \leq x \) so we are done. \( \square \)

4. Prove by induction that

\[
1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}
\]

for every natural number \( n \).