Homework # 4, MAT 498

Presentation due Feb. 12, in written form due Feb. 14

1. Prove that $p(n) < F_n$ for $n \geq 5$, where F_n is the n-th Fibonacci number.

2. Recall that $p(n,k)$ is the number of partitions of n into k parts. Let $p(n,k,D)$ be the number of partitions of n into k distinct parts. Give a simple combinatorial proof that $p(n,k) = p(n + \binom{k}{2}, k, D)$.

3. The logarithmic derivative of a power series $F(x)$ is $\frac{d}{dx} \log F(x) = F'(x)/F(x)$. Let $\sigma(n)$ be the sum of divisors of n, i.e., $\sigma(n) = \sum_{d|n} d$. By logarithmically differentiating the porwer series

$$p(x) = \sum_{n \geq 0} p(n) x^n = \prod_{i \geq 1} (1 - x^i)^{-1},$$

show that

(1). $p'(x)/p(x) = \sum_{n \geq 0} \sigma(n) x^{n-1}$.

(2). $n \cdot p(n) = \sum_{i=1}^{n} \sigma(i) p(n-i)$.