Homework # 2, MAT 498

Presentation due Jan. 29, in written form due Jan. 31

1. Let F_n denotes the nth Fibonacci number. Express the following numbers in terms of the Fibonacci number.

(a). There are n seating positions arranged in a line. Find the number of ways of choosing a subset of these positions, with no two chosen positions consecutive.

(b). If the n positions are arranged around a circle, what is the number of choices?

2. Show the following identities:

$$(a).\ \sum_{k+j=l} \binom{m}{k}\binom{n}{j} = \binom{m+n}{l}$$

$$(b).\ (1-4x)^{-\frac{1}{2}} = \sum_{n\geq 0} \binom{2n}{n} x^n.$$

$$(c).\ \sum_{i=0}^{n} \binom{2i}{i}\binom{2n-2i}{n-i} = 4^n \text{(Hint: use } (b)\text{)}.$$

3. Solve the following recurrence relations:

(a). $f(0)=0, f(1)=1, f(n)=3f(n-1)-2f(n-2)$ for $n \geq 2$.

$$(b).\ f(n) = 1 + \sum_{i=0}^{n-1} f(i), f(0) = 1.$$

4. Evaluate the sum $\sum_{i=1}^{n} \binom{n}{k} k^2$.