Homework # 1, MAT 498

Presentation due Jan. 22, in written form due Jan. 24

1. Give combinatorial proofs of the following identities:

(a). $\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$ (b). $\binom{2n-1}{n} = \sum_{k=0}^{n}\binom{n}{k}\binom{n-1}{k}$

(c). $\binom{3n}{n} = \sum_{i=0}^{n}\binom{n}{i}\binom{2n}{i}$ (d). $\sum_{i=0}^{n} i\binom{n}{i} = n \cdot 2^{n-1}$

2. How many paths are there in the plane from $(0,0)$ to (m,n), if each step in the path is of the form $(1,0)$ and $(0,1)$(this is to say, unit distance due east or north)?

3. We let $[n] = \{1, 2, \cdots, n\}$. Find as simple a solution as possible to the following problem:

(a). How many subsets of the set [10] contains at least one odd number?

(b). How many permutations $\pi : [6] \to [6]$ satisfy $\pi(1) \neq 2$?

(c). There are four men and six women. Each man marries one of the women. In how many ways can this be done?

(d). In how many different ways can the letters of MISSISSIPPI be arranged if the four S's cannot appear consecutively?

4. Show that $(kn)!$ is divisible by $(n!)^k$.

5. Show that $\sum_{d|n} \phi(d) = n$.