UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS
MAT 235 Y — CALCULUS II
FALL–WINTER 2007–2008
ASSIGNMENT #2, DUE ON OCTOBER 25
COVER PAGE

INSTRUCTIONS: Submit this page as your cover page. Use different pages for each of the assigned problems. Staple all your papers and clearly write your name and student number in each of the pages. Show and explain all the steps in each of your solutions. Marks will be deduced for messy or unclear solutions. Late submissions will not be accepted.

NAME: ______________________________ , ______________________
(family name, please print) (given name)

STUDENT NUMBER: ______________________________

SECTION: (please choose one)

SECTION	DAY/TIME	ROOM	INSTRUCTOR	MY SECTION
L–0101	MWF/9	SS–2135	Y. Kim	
L–0201	MWF/2	SS–1073	R. Pujol	
L–0301	MWF/2	RW–110	B. Hovinen	
L–501	R/6-9	LM–161	F. Recio	

PLEASE DO NOT WRITE HERE

Question 1 (out of 3)	Question 2 (out of 3)	Question 3 (out of 3)	Question 4 (out of 3)	Question 5 (out of 3)	TOTAL (out of 15)

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PROBLEMS

DO NOT SUBMIT YOUR SOLUTIONS WITHOUT THE COVER PAGE. READ THE INSTRUCTIONS WRITTEN ON THAT PAGE.

1. Spheres
a) Find the distance between the two spheres S_1 and S_2:

$$S_1 : (x+3)^2 + (y+7)^2 + (z-1)^2 = 9, \qquad S_2 : x^2 + (y+5)^2 + (z-5)^2 = 4.$$

b) Find the distance between the sphere S_1 and the plane P:

$$P: \; x + 2y - 2z = 5.$$

2. Ellipse
a) Consider the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

What is the center and the radius of the osculating circle at the x-intercepts and y-intercepts?

b) Draw the ellipse and its foci for $a = 1$, $b = 2$. Draw the osculating circles at the points of intersection with the x and y axes.

3. Cross-product
a) Let $\mathbf{v}$ be a unit vector in $\mathbb{R}^3$. Let $\mathbf{a}$ be a vector, orthogonal to $\mathbf{v}$. Explain why

$$\mathbf{v} \times (\mathbf{v} \times \mathbf{a}) = -\mathbf{a}.$$

b) Let $\mathbf{v}$ be a unit vector in $\mathbb{R}^3$ and $\mathbf{u}$ any vector in $\mathbb{R}^3$. Explain why

$$\mathbf{v} \times (\mathbf{v} \times (\mathbf{v} \times (\mathbf{v} \times \mathbf{u}))) = -\mathbf{v} \times (\mathbf{v} \times \mathbf{u}).$$

4. Let $\mathbf{r}(t)$ be a curve in $\mathbb{R}^3$. Denote by $\mathbf{v}(t)$ its velocity. Denote by $\theta(t)$ the angle between $\mathbf{r}(t)$ and $\mathbf{v}(t)$.
a) Give an expression for $\frac{d}{dt}|\mathbf{r}(t)|$ in terms of $\mathbf{v}(t)$ and $\theta(t)$ only.

b) If $\theta(t) = \frac{\pi}{2}$ for all t, what can you conclude from question a)?

5. Consider the curve defined by the vector function

$$\mathbf{r}(t) = (\cos t + t \sin t)\,\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + \frac{\sqrt{3}}{2} t^2\,\mathbf{k}.$$

a) Find $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

b) Find the curvature κ.

c) Determine the tangential and normal components of the acceleration; express the acceleration vector $\mathbf{a}(t)$ in terms of $\mathbf{T}$ and $\mathbf{N}$.