MAT 332, Fall 2018. Assignment 3. Due on November 8 (note the change), in class.

**1.** Linearize the following linear system at $(0,0)$ to show that it is an unstable fixed point. Then produce a Maple plot showing that the system has a unique stable limit cycle.

a) Rayleigh system:

$$\begin{cases} x' = y \\ y' = -x - y(\frac{y^2}{3} - 1) \end{cases}$$

a) van der Pol system:

$$\begin{cases} x' = y \\ y' = -x - 5y(x^2 - 1) \end{cases}$$

**2.** Consider the system

$$\begin{cases} x' = -x + 4y \\ y' = -x - y^3 \end{cases}$$

Show that $V(x,y) = x^2 + 4y^2$ is a Lyapunov function for the system. What is the stability of the fixed point $(0,0)$? Are there any periodic orbits? Explain. Illustrate with a Maple plot.

**3.** Consider the system

$$\begin{cases} x' = x(1 - 4x^2 - y^2) - 0.5y(1 + x) \\ y' = y(1 - 4x^2 - y^2) + 2x(1 + x) \end{cases}$$

a) Verify that the origin is an unstable fixed point.

b) Use the Lyapunov function

$$L(x,y) = (1 - 4x^2 - y^2)^2$$

and Poincaré-Bendixson Theorem to show that the system has a limit cycle.

**Hint:** First show that $\frac{d}{dt}L(x(t), y(t)) < 0$ except at the origin and the ellipse $4x^2 + y^2 = 1$. Then draw contours inside and outside the ellipse, on which $L(x,y)$ is constant and positive.

c) Draw a Maple plot to verify that this cycle is the ellipse

$$4x^2 + y^2 = 1.$$