

MAT 332, FALL 2018. ASSIGNMENT 2. DUE ON OCTOBER 18, IN
CLASS.

1. Consider the non-linear system

$$\begin{cases} x' = -y + ax(x^2 + y^2) \\ y' = x + ay(x^2 + y^2) \end{cases}$$

depending on the parameter a .

- a) Linearize the system at the origin and verify that $(0, 0)$ is a centre for the linearized system.
- b) Use a Maple plot for a value $a \neq 0$ to verify that $(0, 0)$ is **not** a centre for the non-linear system.
- c) Re-write the original system in polar coordinates

$$x = r \cos \theta, \quad y = r \sin \theta$$

and simplify. (Hint: the system will take a very simple form if you do this correctly).

d) Finally, using the system in polar coordinates do a study of the trajectories of the system. Identify all possible scenarios for the phase portrait of the system depending on the value of a , and illustrate with phase plots in (x, y) coordinates.

2) Invent your own nonlinear system describing two competing biological species similar to the Lotka-Volterra equations on page 145. Study its behaviour and illustrate with a Maple plot.