

# Variation Of The Triangle Inequality

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The following is a useful variation of the Triangle Inequality.

**Theorem:** Let  $x, y \in \mathbb{R}$ . Then  $|x - y| \geq |x| - |y|$ .

*Proof.* The Triangle Inequality states that  $\forall x, y \in \mathbb{R}, |x + y| \leq |x| + |y|$ .

Let  $z = x + y$ . Then  $x = z - y$ . Replacing all instances of  $x$  in the triangle inequality with  $z - y$ , we get:

$$|(z - y) + y| \leq |z - y| + |y|$$

Rearranging the terms above, we get  $|z| - |y| \leq |z - y|$  or  $|z - y| \geq |z| - |y|$  as desired.  $\square$

Structure:

Assumptions:

Definitions and Results:

Scope: