

- (1) Let M be the Heisenberg group, i.e the group of 3×3 matrices of the form

$$\begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}$$

identified with \mathbb{R}^3 via (x, y, z) coordinates.

Let X, Y, Z be the left invariant vector fields on M with $X(0) = \frac{\partial}{\partial x}, Y(0) = \frac{\partial}{\partial y}, Z(0) = \frac{\partial}{\partial z}$.

Compute the Lie brackets $[X, Y], [X, Z], [Y, Z]$ in two different ways: as originally defined for general vector fields and using the formula $[X, Y] = ad_X(Y)$. Verify that the answers are the same.

- (2) Prove that the Heisenberg group does not admit a biinvariant Riemannian metric.