(1) Prove that if (M^n, g) is a compact manifold without a boundary then all geodesics are infinitely extendable i.e, if $\gamma\colon (-\epsilon, \epsilon) \to M$ is a geodesic then it can be extended to a geodesic defined on $\mathbb{R}$.

(2) Let (M^n, g) be a Riemannian manifold and let ∇ be the Levi-Civita connection for metric g. Let $p \in M$ be any point.

Prove that there exists a coordinate chart x near p such that $\nabla_{\frac{\partial}{\partial x_i}} \frac{\partial}{\partial x_j}(p) = 0$ for all i, j.

Hint: Use the exponential map to define x.