

- (1) Let M^n be a smooth manifold and let ∇ be a connection on M . Let R be the curvature of ∇ , i.e.

$$\mathbb{R}(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z$$

Prove that R is a tensor.

- (2) Let ∇ be a connection on \mathbb{R}^3 given by the formula

$$\nabla_X Y = \nabla_X^{can} Y + X \times Y$$

where \times is the cross product on \mathbb{R}^3 .

- (a) Find the curvature and the torsion of ∇ .
(b) Let $\gamma(t) = (0, 0, t)$ and $v = (1, 0, 0)$. Let $X(t)$ be the parallel vector field along γ with $X(0) = v$.
Find the explicit formula for $X(t)$.