(1) Let M be the Heisenberg group, i.e the group of 3×3 matrices of the form

$$\begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}$$

identified with $\mathbb{R}^3$ via (x, y, z) coordinates.

(a) Consider the left-invariant metric on M which has $g_{ij}(0) = \delta_{ij}$.
   (i) Find $g_{ij}(x, y, z)$
   (ii) Find the volume form generated by g.

(b) Let ∇ be the left invariant connection on M. I.e. let ∇ be defined by $\nabla_X Y = 0$ for any left invariant vector fields X, Y. Find Christoffel symbols of ∇.