

Construct infinitely differentiable functions $M(x, y), N(x, y)$ on $R^2 \setminus (0, 0)$ satisfying

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

and such that there does not exist a function $H(x, y)$ on $R^2 \setminus (0, 0)$ satisfying

$$\frac{\partial H}{\partial x} = M, \frac{\partial H}{\partial y} = N$$