

(1) Consider the IVP

$$\begin{cases} y' = y^2 + 1 \\ y(0) = 0 \end{cases}$$

Find the biggest interval $(-a, a)$ for which the solution is guaranteed to exist by the existence theorem.

(2) Show that the solution of the IVP

$$\begin{cases} y' = \sin(y) + 1 \\ y(t_0) = y_0 \end{cases}$$

exists for all real t .

Hint: Show that the existence interval guaranteed by the existence theorem is the same for all initial conditions t_0, y_0 .

(3) Let $t \rightarrow A(t)$ be a C^1 matrix valued function on \mathbb{R} .

Show that $f(t) = e^{A(t)}$ is continuous in t .

Hint: Use continuous dependence of solutions of IVPs on parameters.