(1) Find the general solution of the following system

$$\begin{cases} y_1' = 5y_1 + 3y_2 + t \\ y_2' = -6y_1 - 4y_2 \end{cases}$$

(2) Solve the following IVP

$$\begin{cases} y_1' = 2y_1 - y_2 + e^{2t} \\ y_2' = y_1 + 2y_2 - e^{2t} \\ y_1(0) = 0 \\ y_2(0) = 1 \end{cases}$$

(3) Prove the Uniqueness theorem for a non-autonomous system

$$\begin{cases} y' = f(t, y) \\ y(t_0) = y_0 \end{cases}$$

where $f$ is a $C^1$ function in $(t, y)$.

*Hint:* Reduce the problem to an autonomous system by introducing an extra variable.

(4) Let $y(t)$ be the solution of the following IVP:

$$\begin{cases} y' = y \sin^2 y \\ y(0) = 1 \end{cases}$$

a) Prove that $y(t) \geq 0$ for $t > 0$.
*Hint:* show that $y(t)$ is nondecreasing for $t > 0$.

b) Prove that $y(t) \leq e^t$ for $t \geq 0$.
*Hint:* Show that $y(t)$ satisfies a differential inequality of the form $y' \leq ay$ for some $a > 0$ and use it to estimate $y(t)$ from above.