(1) Let A be the following matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ -2 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & 1 \end{pmatrix}$$

Find a basis of the solution space for the system $y' = Ay$ where y is a column vector.

(2) Show that e^A is orthogonal if A is real and skew-symmetric.

(3) Show that if $e^{tA}e^{tB} = e^{tB}e^{tA}$ for any real t then $AB = BA$.

Hint: Differentiate!