(1) Let U be an orthogonal $n \times n$ matrix with determinant 1 (i.e. $U \in SO(n)$.

Prove that there exists a skew-symmetric real matrix A such that $U = e^A$.