Term Test 3 Practice Test 2

(1) Give the following definitions
   (a) a k-tensor on a vector space V.
   (b) A C^r-manifold without a boundary in $\mathbb{R}^n$.

(2) Let e_1, e_2, e_3, e_4 be a basis of a 4-dimensional space V.
   Let $\omega = Alt(e_1^* \otimes e_2^* + e_3^* \otimes e_4^*)$ and $\eta = 2e_2^* + e_3^*$.
   Find $(\omega \wedge \eta)(e_2, e_3, e_4)$.

(3) Let v be an n-dimensional vector space. Let $k \geq 1$. Find all k-tensors T on V such that both T and $|T|$ are tensors.

(4) Let $M \subset \mathbb{R}^3$ be given by $\{x^2 + y^2 - z^2 = 0\} \cap \{x + 2y - z = 1\}$.
   Show that M is a manifold and compute its dimension.

(5) Let $U \subset \mathbb{R}^2$ be given by $\{0 < x^2 + 4y^2 < 1\}$ and $f(x, y) = \frac{1}{\sqrt{x^2+4y^2}}$.
   Determine if $\int_U^{ext} f$ exists and if it does compute it.

(6) Let $v_1 = (1, 1, 0), v_2 = (-1, 0, 1), v_3 = (1, 1, 1)$ and $w_1 = (0, 2, 0), w_2 = (1, 1, 0), w_3 = (-2, 1, 3)$ be two bases of $\mathbb{R}^3$. Do (v_1, v_2, v_3) and (w_1, w_2, w_3) have the same orientation?

(7) let $M \subset \mathbb{R}^n$ be a manifold with boundary and $N \subset \mathbb{R}^m$ be a manifold without boundary. Prove that $M \times N \subset \mathbb{R}^{n+m}$ is a manifold with boundary.