MAT 257Y **Term Test 2 Practice test 2**

(1) (20 pts) Let $F(x,y)$ be given by the formula

$$F(x,y) = \int_0^y e^x \cos(t^2 x)dt$$

(a) Show that F is C^1 on $\mathbb{R}^2$.

(b) Let $c = F(0,2)$. Compute c and prove that near $(0,2)$ the level set $\{F(x,y) = c\}$ can be written as a graph of a differentiable function $y = g(x)$ and find $g'(0)$.

(2) (10 pts) Let $U \subset \mathbb{R}^n$ be open and $f\colon U \to \mathbb{R}^n$ be C^1 such that $\det([df_p]]) \neq 0$ for any $p \in U$. Show that $f(U)$ is open.

(3) (10 pts) Let $Q \subset \mathbb{R}^n$ be a rectangle and let $f, g\colon Q \to \mathbb{R}$ be integrable. Prove that $f \cdot g$ is integrable over Q.

(4) (15 pts) Mark True or False. **If true, give a proof. If false, give a counterexample.**

(a) Let (X,d) be a metric space. If $f\colon X \to \mathbb{R}$ satisfies $|f(x) - f(y)| \leq 10 \cdot d(x,y)$ for any $x, y \in X$ then f is uniformly continuous.

(b) Let $A \subset \mathbb{R}^n$ be a rectangle and $S \subset A$ be a subset of measure 0 then S is rectifiable.

(c) Let $A \subset \mathbb{R}^n$ be a rectangle and $f\colon A \to \mathbb{R}$ be continuous except at finitely many points. Then f is integrable over A.

(5) (15 pts) Let A be a rectangle and $f\colon A \to \mathbb{R}$ be integrable such that f vanishes except on a set $S \subset A$ of measure 0. Prove that $\int_A f = 0$.

Hint: Show that $\overline{\int}_A f \geq 0$ and $\underline{\int}_A f \leq 0$.

(6) (15 pts) Let $S = \{(x,y) \in \mathbb{R}^2 |$ such that $y^2 \leq x \leq 4\}$. Let $f(x,y) = y$.

Prove that f is integrable over S and compute $\int_S f$.

(7) (15 pts) Let S be a compact rectifiable set. Let $S \subset \cup_{i=1}^{\infty} Q_i$ for a countable collection of rectangles Q_i. Prove that $\mathrm{vol}(S) \leq \sum_{i=1}^{\infty} \mathrm{vol}(Q_i)$.

Hint: Reduce the problem to the case of covering by open rectangles.