MAT 257Y **Practice Term Test 1**

(1) Find the partial derivatives of the following functions
  (a) $F(x, y) = f(g(x)k(y), h(x) + 2k(y))$
  (b) $f(x, y, z) = \sin(x \sin(y \sin z))$
  (c) $f(x, y, z) = x^{yz^2}$

(2) Give an example of a nonempty subset A of $\mathbb{R}$ such that the set of limit points of A in $\mathbb{R}$ is the same as the set of boundary points of A.

(3) Let $A, B \subset \mathbb{R}^n$ be compact.
  Prove that the set $A + B = \{a + b | a \in A, b \in B\}$ is compact.

(4) Show that the intersection of an arbitrary collection of closed sets is closed.

(5) Prove that $f\colon \mathbb{R}^n \to \mathbb{R}^m$ is continuous if and only if $f^{-1}(A)$ is closed for any closed $A \subset \mathbb{R}^m$.

(6) Let $GL(n, R)$ be the set of all $n \times n$ invertible matrices identified with $\mathbb{R}^{n^2}$.
  Show that $GL(n, R)$ is open in $\mathbb{R}^{n^2}$.

(7) Let $f = (f_1, f_2)\colon \mathbb{R}^2 \to \mathbb{R}^2$ be given by the formula $f_1(x, y) = x + y + y^3 + 1$, $f_2(x, y) = xe^y + 2$
  Show that there is an open set U containing $(0, 0)$ such that $f\colon U \to f(U)$ is a bijection and f^{-1} is differentiable on $f(U)$ and compute $df^{-1}_{(1,2)}$.

(8) Let $f(x, y) = x^y$ be defined on $U = \{(x, y) | x > 0\}$.
  Verify that

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{\partial^2 f}{\partial y \partial x}(x, y)$$

  for any $(x, y) \in U$.