(1) Let $f\colon [0.1] \to \mathbb{R}$ be given by $f(x) = \sqrt{x}$.

Using the definition prove that f is uniformly continuous but not Lipschitz on $[0,1]$.

(2) (a) Let $f(x) = \int_{x^2}^{3x} \sqrt{t^3 + x^3}dt$ Find the expression for $f'(x)$.
You **DO NOT** need to evaluate the integral in that expression.

(b) Let $f(x) = \int_0^{h(x)} (g(x,t))^4 dt$
where h and g are C^1.
Find the formula for $f'(x)$.

(c) Let $f(x) = \int_a^{\int_b^x g(x,y)dy} g(x,y)dy$ where g is C^1.
Find the formula for $f'(x)$.

(3) Let $S \subset \mathbb{R}^n$ be bounded and let $f\colon S \to \mathbb{R}$ be bounded. Show that $\int_S f$ is well defined. That is, let

$$f_S(x) = \begin{cases} f(x) \text{ if } x \in S \\ 0 \text{ if } x \notin S \end{cases}$$

Take a rectangle A containing S. Suppose $\int_A f_s$ exists.

prove that for any other rectangle A' containing S the integral $\int_{A'} f_S$ also exists and $\int_{A'} f_S = \int_A f_S$.

(4) Let $S \subset \mathbb{R}^n$ be a rectifiable set and let $f\colon S \to \mathbb{R}$ be continuous and bounded. Prove that f is integrable over S.

(5) Consider the modified Cantor set S on $[0,1]$ constructed as follows. Let S_1 be obtained from $[0,1]$ by removing the open interval $(\frac{1}{2} - \frac{1}{2\cdot 5}, \frac{1}{2} + \frac{1}{2\cdot 5})$ of length $\frac{1}{5}$. Note that S_1 is a union of two closed intervals $I_1 = [0, \frac{1}{2} - \frac{1}{2\cdot 5}]$ and $I_2 = [\frac{1}{2} + \frac{1}{2\cdot 5}, 1]$. Let S_2 be obtained from S_1 by further removing the "middle" open intervals of length $\frac{1}{5^2}$ from I_1 and I_2 etc. Let $S = \cap_{i=1}^{\infty} S_i$ be the Cantor set.

(a) Show that $S = bd(S)$;

(b) Show that S is not rectifiable.
Hint: Argue by contradiction. Assume that S has measure zero. Then show that $[0,1]$ can be covered by countably many intervals with total length < 1.