(1) Show that the union of finitely many compact sets is compact.

(2) Using only the definition of differentiability prove that if $f, g\colon \mathbb{R}^n \to \mathbb{R}^m$ are differentiable at $p \in \mathbb{R}^n$ then $f+g$ is also differentiable at p and $d(f+g)_p = df_p + dg_p$.

(3) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = xy$.

Prove that f is differentiable everywhere and compute df_p for $p = (a,b)$.

(4) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = \begin{cases} \sqrt{|xy|} \text{ if } x \geq 0 \\ -\sqrt{|xy|} \text{ if } x < 0 \end{cases}$$

Show that $D_h f((0,0))$ exists for any $h \in \mathbb{R}^2$ but f is not differentiable at $(0,0)$.

(5) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = xy + y^2$$

Let $p = (1,2)$. Prove that f is differentiable at p and compute df_p.

(6) Finish the proof of a theorem from class: Let $f = (f_1, \ldots, f_k)\colon \mathbb{R}^n \to \mathbb{R}^k$. Then f is differentiable at p if each f_i is differentiable at p.

Extra Credit Problem (to be written up and submitted separately)

Prove that any two norms on $\mathbb{R}^n$ define the same open sets.