(1) Let $\omega_m = \sum_{i=1}^{n} (-1)^{i-1} \frac{x_i}{|x|^m} dx_1 \wedge \ldots \wedge \hat{dx_i} \wedge \ldots \wedge dx_n$ be an n-form on $\mathbb{R}^{n-1}$

Prove that for an appropriate choice of m ω is closed.

(2) Prove that the unit sphere $S^{n-1} \subset \mathbb{R}^n \backslash \{0\}$ is not a boundary of a compact n manifold. *Hint:* Use Stokes' Theorem.

(3) Let $M^k \subset \mathbb{R}^n$ be an oriented compact manifold and let dV be the volume form on M. Prove that $\int_M dV > 0$.

(4) Let $V \subset \mathbb{R}^3$ be a 2-dimensional subspace. Let $n \in R^3$ be a unit normal vector to V. It induces orientation μ on V. Let ω be the volume tensor on V induced by this orientation and the standard inner product on V.

Prove that for any $u, v \in V$ we have $\omega(u, v) = \det \begin{pmatrix} u \\ v \\ n \end{pmatrix}$ *Hint:* Since V is 2-dimensional and ω is an alternating 2-tensor on V it's enough to check the equality on any positive orthonormal basis e_1, e_2 of V.

(5) Let $M^k \subset \mathbb{R}^n$ be a k-manifold.

Prove that M is orientable if and only if there exists a nowhere zero smooth k form on M.

(6) Let $M \subset \mathbb{R}^n$ be a 1-manifold in $\mathbb{R}^n$. Let $U \subset \mathbb{R}^n$ be open and let $c\colon (0,1) \to M \cap U$ be a local parameterization coming from the definition of a manifold. Prove that $\int_{M \cap U} dV = \int_0^1 |c'(t)| dt$.