(1) Let $U \subset R^n, V \subset R^m$ be open. Show that $U \times V \subset R^{n+m}$ is open.
(2) Let $A \subset R^n, B \subset R^m$ be closed. Show that $A \times B \subset R^{n+m}$ is closed.
(3) Let X be a metric space. Let $A \subset X$ be a subset of X. Prove that $Lim(A)$ is closed.
(4) Prove that the definition of the interior of A given in class is equivalent to the definition given in the book. In other words. Let X be a metric space. Let $A \subset X$ be a subset of X. In class we defined $int(A)$ to be the set of points $p \in X$ such that there is $\varepsilon > 0$ such that $B_\varepsilon(p) \subset A$.

    Prove that with this definition $int(A)$ is the union of all open sets contained in A.
(5) Let $f\colon R^n \to R^m$ be a continuous map. Is it true that image of every closed set under f is closed? Prove or give a counterexample.
(6) Let $f = (f_1, \dots f_n)\colon X \to R^n$. Prove that f is continuous if and only if all f_i are continuous.
(7) Find $bd(A), Lim(A)$ and $Cl(A)$ for the following sets.
    (a) $A = \{0 < x^2 + y^2 \leq 1\} \subset \mathbb{R}^2$.
    (b) $A = (0,1) \times \{0\} \subset \mathbb{R}^2$.
    (c) $A = \{(x,y) \in \mathbb{R}^2 |$ such that $x > 0, y < \sin(1/x)\} \subset \mathbb{R}^2$.
(8) Let $f\colon X \to Y$ and $g\colon Y \to Z$ where X, Y, Z are metric spaces. Suppose f is continuous at p and g is continuous at $f(p)$. Using only the definition prove that $g \circ f$ is continuous at p.
(9) Let $A = \{0, 1, 1/2, 1/3, \dots, 1/n, \dots\}$. Using only the definition of compactness prove that A is compact as a subset of $\mathbb{R}$.
(10) Let $A = \mathbb{Q} \cap [0,1] \subset \mathbb{R}$. Give an example of an open cover of A which has no finite subcover.

Extra Credit Problem (to be written up and submitted separately)

Give an example of a nonempty set $A \subset \mathbb{R}$ such that $A = br(A) = Lim(A) = Cl(A)$.