

- (1) Let  $M = S^n \subset \mathbb{R}^{n+1}$  be the unit sphere around 0. Let  $S_{eq}$  be the equator in  $M$ , i.e.  $S_{eq} = M \cap (\mathbb{R}^n \times \{0\})$ .

Prove that  $S$  has measure zero in  $M$ .

- (2) Let  $M$  be the cylinder  $\{(x, y, z) \mid x^2 + y^2 = 1 \text{ and } 0 \leq z \leq 1\}$  in  $\mathbb{R}^3$ . Let  $\omega = z dx$ . Fix an orientation on  $M$  such that the  $e_2 = (0, 1, 0)$ ,  $e_3 = (0, 0, 1)$  give a positive basis of  $T_p M$  for  $p = (1, 0, 0)$ .

Compute  $\int_M d\omega$  and  $\int_{\partial M} \omega$  and verify that they are equal.

- (3) Let  $M$  be the solid torus obtained by rotating the disk  $(x-2)^2 + z^2 \leq 1$  in the  $xz$ -plane around the  $z$ -axis with the canonical orientation coming from  $\mathbb{R}^3$ .

Let  $\omega = \frac{z}{\sqrt{x^2+y^2}} dx \wedge dy$ .

Find  $\int_{\partial M} \omega$ .

*Hint:* Use Stokes' Theorem and cylindrical coordinates.