(1) Let $\sigma \in S_n$. Define P_σ to be the $n \times n$ matrix with $(P_\sigma)_{ij} = \delta_{i,\sigma(j)}$.

Prove that the map $\sigma \mapsto P_\sigma$ is a homomorphism from S_n to $O(n)$. What is the kernel of this homomorphism?

(2) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 1 & 5 & 4 & 2 \end{pmatrix}$

Find $sign(\sigma)$.

(3) Let A be an $n \times n$ matrix.

Prove that $\det A = \sum_{\sigma \in S_n} sign(\sigma) A_{1\sigma(1)} \cdot \ldots \cdot A_{n\sigma(n)}$.

(4) Let T be a k-tensor on V. Prove that T is alternating if and only if $T(v_1, \ldots v_k) = 0$ for any $v_1, \ldots v_k$ such that $v_i = v_j$ for some $i \neq j$.

(5) let $\sigma \in S_{k+l}$ be the following permutation

$$\begin{pmatrix} 1 & \ldots & k & k+1 & \ldots & k+l \\ l+1 & \ldots & l+k & 1 & \ldots & l \end{pmatrix}$$

Prove that $sign(\sigma) = (-1)^{kl}$.