(1) Finish the proof of the theorem from class: Let $M^k \subset \mathbb{R}^n$ be a manifold with boundary. Let $V_1, V_2 \subset H^k$ be open subsets and let $f_i \colon V_i \to M$ (with $i = 1, 2$) be local parameterizations from the definition of a manifold such that $f_i(q_i) = p$. Let $g = f_2^{-1} \circ f_1 \colon V_1' \to \mathbb{R}^k$ where $V_1' = f_1^{-1}(f_2(V_2))$. Prove that g is differentiable at q_1 and dg_{q_1} is invertible.

(2) Let $M^k \subset \mathbb{R}^n$ be a manifold with boundary. Prove that $\partial M \subset M$ is closed in M. Is it true that ∂M must be closed in $\mathbb{R}^n$?

(3) Prove that $M = (x, y) \in \mathbb{R}^2$ such that $y \geq \sqrt{|x|}$ is **not** a manifold with boundary.

Extra Credit: Let $M_r(m, n) \subset \mathbb{R}^{mn}$ be the set of all $m \times n$ matrices of $rank = r$ where $r \leq min(m, n)$. Prove that $M_r(m, n)$ is a manifold of dimension $(m - r)(n - r)$.

Hint: Use that a matrix has $rank = r$ if and only if it ca be reduced by elementary row operations to a matrix whose last $(m - r)$ rows are zero and the first r rows are linearly independent. Also use that elementary row operations are given by multiplications by invertible matrices.