(1) Let $S \subset \mathbb{R}^n$ be a closed subset. Let $f\colon S \to \mathbb{R}^m$ be a map. Prove that f is C^r if and only f can be extended to a C^r map $F\colon \mathbb{R}^n \to \mathbb{R}^m$.

Hint: Use partition of unity.

(2) Show that a subset $M \subset \mathbb{R}^n$ is an n-manifold without a boundary if and only if M is open.

(3) Prove that the union of xy plane and xz plane in $\mathbb{R}^3$ is not a C^1-manifold.

Hint: Use that a 2-manifold is locally given as a level set $\{f = c\}$ for some C^1 function f and a regular value c of f.

(4) Let $M^k \subset \mathbb{R}^n$ be a C^r manifold without boundary. let $g\colon M \to \mathbb{R}$ be a function.

Prove that g is C^r if and only if for any $p \in M$ there exists a parametrization $f\colon V \to U$ where $V \subset \mathbb{R}^k$ is open and $U \subset M$ is open satisfying the definition of a manifold such that $g \circ f$ is C^r.

(5) Let $n > 1$ and Let M be the set of $n \times n$ matrices A with $\det(A) = 1, tr(A) = 0$ considered as a subset of the space of all $n \times n$ matrices which is identified with $\mathbb{R}^{n^2}$. Prove that M is a manifold and compute its dimension.