(1) Let $U \subset \mathbb{R}^n$ be open and let $f\colon U \to \mathbb{R}^m$ be C^1 where $m > n$. Prove that $f(U)$ has measure zero.

(2) Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Write A as a composition of primitive linear diffeomorphisms.

(3) Let $f\colon R^2 \to R$ be continuous and odd. Recall that f is called odd if $f(-x) = -f(x)$ for any $x \in R^2$.

Prove that $\int_{x^2+y^2<1} f = 0$.

(4) Let $\bar{B}(0,R)$ be the closed ball of radius R in R^n. Prove that $\mathrm{vol}\bar{\mathrm{B}}(0,\mathrm{R}) = \mathrm{R}^{\mathrm{n}}\mathrm{vol}\bar{\mathrm{B}}(0,1)$.

(5) Let U be a rectifiable open set in the xz plane lying in the halfplane $x > 0$. Let V be the solid in R^3 obtained by rotating U around the z axis.

Prove that $\mathrm{volV} = \int_{\mathrm{U}} 2\pi\mathrm{x}$.

Hint: use cylindrical coordinates in R^3:

$x = r\cos\theta, y = r\sin\theta, z = z.$

(6) Prove that $\int_{(-\infty+\infty)}^{ext} e^{-x^2} dx = \sqrt{\pi}$.

Hint: Show that $(\int_{(-\infty+\infty)}^{ext} e^{-x^2} dx)(\int_{(-\infty+\infty)}^{ext} e^{-y^2} dy) = \pi$