(1) Let A, B be $n \times n$ real matrices. Define $\langle A, B\rangle = tr(A \cdot B^t)$.

Prove that this defines an inner product on the space of all $n \times n$ matrices.

(2) Let $\{C_i\}_{i\in I}$ be a family of subsets in a set X. Prove that

$$X\backslash(\cup_i C_i) = \cap_i (X\backslash C_i)$$

(3) Show that the norm $||\cdot||_\infty$ on $\mathbb{R}^n$ satisfies the triangle inequality

$$||x + y||_\infty \leq ||x||_\infty + ||y||_\infty$$

for any $x, y \in \mathbb{R}^n$.

(4) Show that the norms $||\cdot||$ and $||\cdot||_\infty$ on $\mathbb{R}^n$ satisfy

$$||x||_\infty \leq ||x|| \leq \sqrt{n} \cdot ||x||_\infty$$

for any $x \in \mathbb{R}^n$.

(5) Prove that metrics coming from $||\cdot||$ and $||\cdot||_\infty$ on $\mathbb{R}^n$ define the same open sets.

Hint: Use Problem (4).

(6) Show that interior of any set is an open set.

(7) Prove that a set $A \subset R^n$ is closed if and only if it contains all its boundary points.

Extra Credit Problem (to be written up and submitted separately)

Suppose $v_1, \ldots v_{k+1}$ are nonzero vectors in R^n such that $\angle v_i v_j > \pi/2$ for any $i \neq j$.

Show that $v_1, \ldots v_k$ are linearly independent.