(1) Prove the the following maps are submersions
   (a) the map $f\colon \mathbb{R}^n\backslash\{0\} \to \mathbb{S}^{n-1}$ given by $f(x) = \frac{x}{|x|}$;
   (b) the map $g\colon \mathbb{C}^{n+1}\backslash\{0\} \to \mathbb{CP}^n$ given by $g(z_0, \dots, z_n) = [z_0 : \dots : z_n]$;
   (c) the map $h\colon \mathbb{S}^{2n+1} \to \mathbb{CP}^n$ given by $h(z_0, \dots, z_n) = [z_0 : \dots : z_n]$ where $\mathbb{S}^{2n+1}$ is the unit sphere in $\mathbb{C}^{n+1}$.

(2) Show that $\mathbb{CP}^1$ is diffeomorphic to $\mathbb{S}^2$.

(3) Let $\phi\colon \mathbb{R} \to S^1$ be given by $\phi(t) = (\cos t, \sin t)$.
   (a) verify that $\phi|_{(a,b)}\colon (a, b) \to \mathbb{S}^1$ is a diffeomrophism onto its image whenever $b - a < 2\pi$
   (b) Show that if $\phi(t_1) = \phi(t_2)$ then $d\phi_{t_1}(\frac{\partial}{\partial t}) = d\phi_{t_2}(\frac{\partial}{\partial t})$ and therefore, for any $z \in \mathbb{S}^1$ we have a well defined vector $\frac{\partial}{\partial t}|_z \in T_z\mathbb{S}^1$.
   (c) Show that the map $F\colon \mathbb{S}^1 \times \mathbb{R} \to TS^1$ given by $F(z, \lambda) = (z, \lambda\frac{\partial}{\partial t}|_z)$ is a diffeomorphism.