(1) Let $n \geq 1$ and let M^n be a closed (i.e compact and with no boundary) oriented manifold. Let $f\colon M \to \mathbb{S}^n$ be a smooth map such that $\int_M f^*\omega \neq 0$ for some $\omega \in \Omega^n(S^n)$.

   Prove that f is onto.

   Hint: Show that if f is not onto then it's homotopic to a constant map.

(2) Let (M^n, g) be a compact orientable Riemannian manifold with orientation ϵ. Let $d\,\mathrm{vol}_g^\epsilon$ be the volume form induced by the orientation ϵ and the metric g. Let $f\colon M \to \mathbb{R}$ be a smooth function.

   Prove that $\int_M f d\,\mathrm{vol}_g^\epsilon$ does not depend on the choice of the orientation ϵ. I.e. given two orientations ϵ_1, ϵ_2 on M we have that

$$\int_{M,\epsilon_1} f d\,\mathrm{vol}_g^{\epsilon_1} = \int_{M,\epsilon_2} f d\,\mathrm{vol}_g^{\epsilon_2}$$