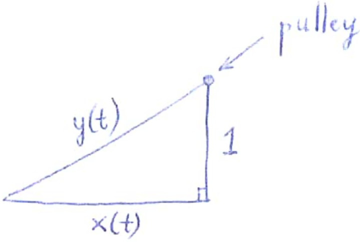
Related Rates

Problem 1



Let $x(t)$ be the distance from the boat to the dock
$y(t)$ be the distance from the boat to the pulley

At a particular time t we have

$$x^2 + 1^2 = y^2 \quad \text{(Pythagorean Theorem)}$$

Differentiate with respect to t we obtain

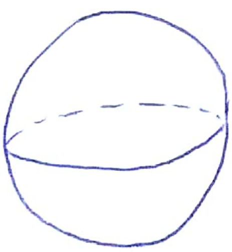
$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

When $x = 8$, we get $y = \sqrt{8^2+1^2} = \sqrt{65}$. Therefore

$$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt} = \frac{\sqrt{65}}{8}\cdot 1 = \frac{\sqrt{65}}{8} \ (m/s),$$

where $\frac{dy}{dt} = 1$ by assumption. $\square$

Problem 2



Let $r(t)$ be the radius of the sphere.

At a particular time t the volume of the sphere is given by

$$V(t) = \frac{4}{3}\pi r^3(t).$$

Thus

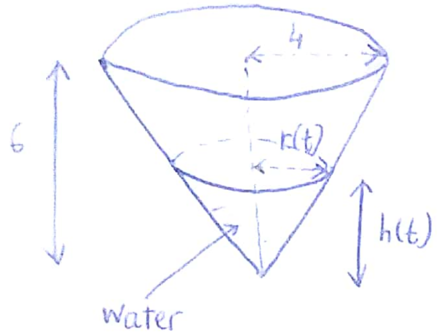
$$V'(t) = 4\pi r^2(t) r'(t).$$

We know that $r'(t) = 4$ and when the diameter is 80, the radius is 40 It follows that the volume is increasing at

$$V' = 4\pi\, 40^2 \cdot 4 = 25{,}600\pi \ (mm^3/s),$$

as required. $\square$

Problem 3

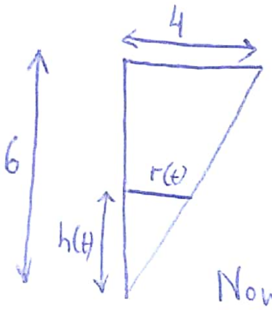


Let $h(t)$ be the ~~radius~~ height of the water and $r(t)$ be the radius of the surface at time t.

The volume of the water at time t is given by

$$V(t) = \frac{1}{3}\pi h r^2.$$

Using similar triangles we obtain



$$\frac{r}{4} = \frac{h}{6} \Rightarrow r = \frac{2}{3}h.$$

Thus we can rewrite the volume as

$$V(t) = \frac{1}{3}\pi h\left(\frac{2}{3}h\right)^2 = \frac{4}{27}\pi h^3.$$

Now let c be the constant rate at which water is being pumped into the tank. Then

$$V'(t) = c - 10{,}000$$

On the other hand,

$$V'(t) = \frac{4}{9}\pi h^2 h'(t).$$

When $h = 200$, we know that $h' = 20$. Therefore,

$$c - 10{,}000 = \frac{4}{9}\pi (200)^2\, 20.$$

Thus $c = 10{,}000 + \dfrac{3{,}200{,}000}{9}\pi \quad (cm^3/s)$.

$\square$