**SOLUTIONS -First QUIZ, MAT244**

**1.** Consider the equation $y'(t) - 3y(t) = te^{3t}$, for $t \in \mathbf{R}$.
a) A multiplier can be computed by

$$\mu(t) = e^{\int(-3)dt} = e^{-3t}.$$

b) After multiplication by integrating factor the equation becomes

$$\frac{d}{dt}\left(e^{-3t}y(t)\right) = t.$$

Integrating in $t$ we get $e^{-3t}y(t) = t^2/2 + C,\ C \in \mathbf{R}$, from where the general solution is

$$y(t) = e^{3t}\left(t^2/2 + C\right),\ C \in \mathbf{R}.$$

c) Let $t_0$ be a point where it touches the horizontal axis. Then we have both $y'(t_0) = 0$ (being tangent) and $y(t_0) = 0$ (being on the axis). From the equation evaluated at $t_0$ we have $0 = t_0e^{3t_0}$ which yields $t_0 = 0$. Therefore we look for the special solution which has $y(0) = 0$. Solving for the constant, we get $C = 0$ and then $y(t) = t^2e^{3t}/2$.

**2.** a) For $x > 0$, divide both sides by $x^2$ to get the standard form of a homogeneous equation

$$2y'(x) = \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right) + 1.$$

substitute $y(x) = xu(x)$ and $y' = u + xu'$ to get the new equation for $2xu'(x) = u^2 + 1$ or

$$\frac{2u'}{u^2+1} = \frac{1}{x}$$

or, applying the chain rule in the left hand side, we get

$$2\frac{d}{dx}(\arctan u(x)) = \frac{1}{x}.$$

Integrating we get $2\arctan u = \ln(x) + C,\ C \in \mathbf{R}$, or

$$u(x) = \tan\left(\frac{1}{2}\ln(x) + C\right),\ C \in \mathbf{R}.$$

Therefore

$$y(x) = xu(x) = x\tan(\frac{1}{2}\ln(x) + C),\ C \in \mathbf{R}.$$

b) $-1 = y(1) = \tan(C)$. For any integer $k$, $C = -\pi/4 + k\pi$ would do. However, for all such $C$ we get

$$y(x) = x\tan\left(\frac{1}{2}\ln(x) - \frac{\pi}{4}\right).$$

c) We need an interval which contains 1 and where $y$ is defined, i.e.

$$-\frac{\pi}{2} < \frac{1}{2}\ln(x) - \frac{\pi}{4} < \frac{\pi}{2}.$$

Solving for $x$ we get

$$e^{-\frac{3\pi}{4}} < x < e^{\frac{3\pi}{4}}.$$