

# Symplectic geometry seminar

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## Projective completions of affine varieties via degree-like functions

Given a ‘degree like’ function  $\delta$ , i.e. a map of the coordinate ring  $A$  of an affine variety  $X$  to integers such that  $\delta(f + g) \leq \max\{\delta(f), \delta(g)\}$  (with strict inequality implying  $\delta(f) = \delta(g)$ ) and  $\delta(fg) \leq \delta(f) + \delta(g)$ , one can associate to it a projective completion of  $X^\delta$  of  $X$ . We show that the ideal  $I_\infty$  of the hypersurface of ‘points at infinity’ is radical iff  $\delta$  is the maximum of finitely many semidegrees (i.e. degree like functions  $\delta'$  such that  $\delta'(fg) = \delta'(f) + \delta'(g)$  for all  $f, g$ ), which are then in a 1-1 correspondence with the prime components of  $I_\infty$ .

Given a projective completion  $Z$  of  $X$  determined by a degree like function  $\delta$ , we define a ‘normalized’ degree like function  $\bar{\delta}$  which is a maximum of finitely many semidegrees. When  $X$  is normal, the corresponding completion is isomorphic to the normalization of  $Z$ . The construction of  $\bar{\delta}$  from  $\delta$  generalizes the construction of the normal toric completion  $X_P$  of  $(C^*)^n$  determined by a convex integral polytope  $P$  from the toric variety determined by an arbitrary finite subset  $S$  of integral points in  $P$  whose convex hull is  $P$ . I will describe this construction and the properties of toric completions these are so far known to preserve.