Abstract:

Let G be a real Lie group acting on a manifold M, let $\mathfrak{g}$ be the Lie algebra of G, and let $\alpha(X)$ be an equivariantly closed form on M depending on $X \in \mathfrak{g}$. For $X \in \mathfrak{g}$, we denote by VF_X the vector field on M induced by the infinitesimal action of X. Then the Berline-Vergne localization formula says that the integral of $\alpha(X)$ can be expressed as a sum over the set of zeroes of VF_X of certain *local* quantities of M and α:

$$\int_M \alpha(X) = \sum_{\substack{p \in M, \\ p \text{ is a zero of } VF_X}} \text{local invariant of } M \text{ and } \alpha \text{ at } p.$$

This formula plays a central role in symplectic geometry and many other branches of mathematics.

For compact groups it was proved by N.Berline and M.Vergne and independently by M.Atiyah and R.Bott more than twenty years ago. In this talk I will explain a version of this localization formula for non-compact group actions.