Welcome back to MAT137- Section L5101

- Assignment #5 due on Dec 20.
- Watch videos 7.1 and 7.2 for Monday, January 11.
- HAPPY HOLIDAYS! :)

Let's get started!!

Today's videos: None!!
Today's topic: Curve sketching

Any question from previous class?

Monotonicity and concavity

Let $f(x) = xe^{-x^2/2}$.

1. Find the intervals where f is increasing or decreasing, and its local extrema.
2. Find the intervals where f is concave up or concave down, and its inflection points.
3. Calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
4. Using this information, sketch the graph of f.

Fractional exponents

Let $h(x) = \dfrac{x^{2/3}}{(x-1)^{2/3}}$. Its first two derviatives are

$$h'(x) = \frac{-2}{3x^{1/3}(x-1)^{5/3}} \qquad h''(x) = \frac{2(6x-1)}{9x^{4/3}(x-1)^{8/3}}$$

1. Find all asymptotes of h
2. Study the monotonicity of h and local extrema
3. Study the concavity of h and inflection points
4. With this information, sketch the graph of h

Hyperbolic tangent

The function tanh, defined by

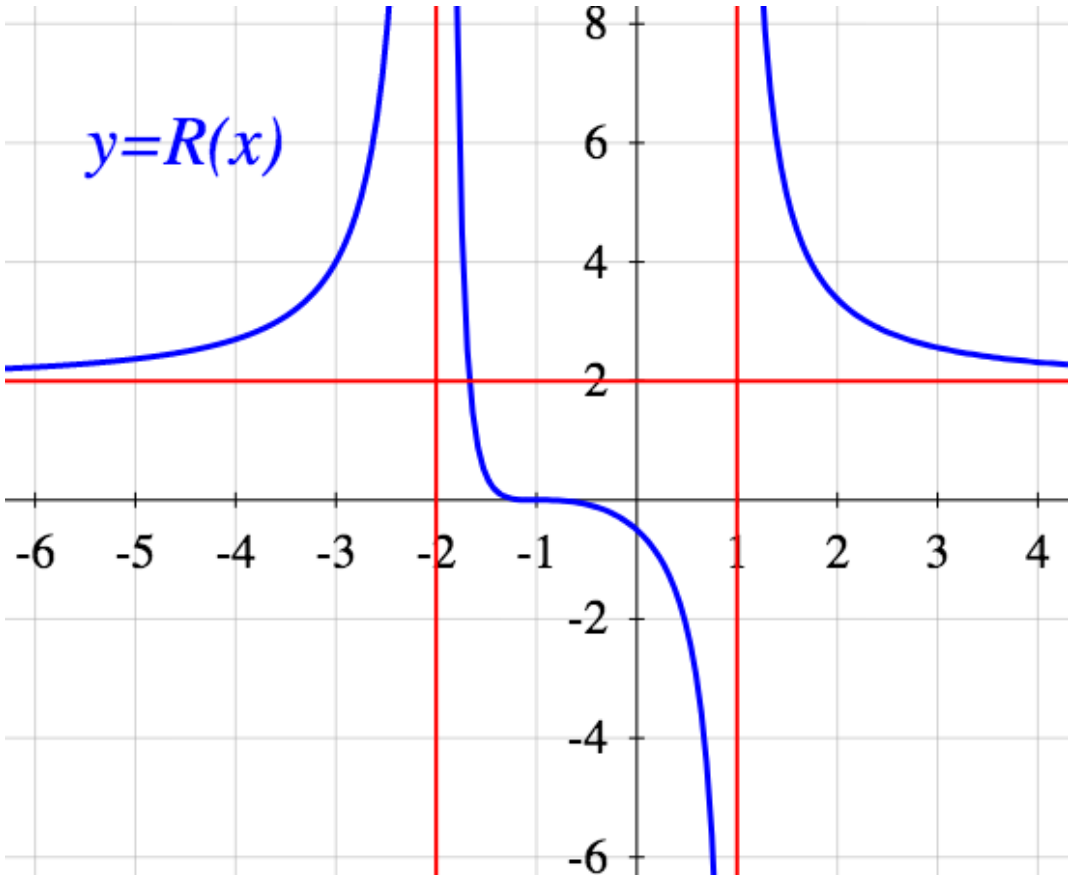
$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}},$$

is called the "hyperbolic tangent".

1. Find its two asymptotes
2. Study its monotonicity
3. Study its concavity
4. With this information, sketch its graph.

Backwards graphing

R is a rational function (a quotient of polynomials). Find its equation.



Suggestion: Play around with Desmos.

Unusual examples

Construct three functions f, g, and h.

1. f has domain at least $(0, \infty)$, is continuous, is always concave up, and satisfies $\lim_{x \to \infty} f(x) = -\infty$

2. g has domain $\mathbb{R}$, is continuous, has a local minimum at $x = 0$, and has an inflection point also at $x = 0$.

3. h has domain $\mathbb{R}$, is differentiable, is strictly increasing. In addition, h' is periodic with period 2, and h' is not constant.