Welcome back to MAT137- Section L5101

- Assignment #4 due on Nov 26.
- Test 2 opens on Dec 4
- Assignment #5 due on Dec 20.
- **Next class: MVT**
  - **Watch videos 5.7, 5.8, 5.9**

Let's get started!!

Today's videos: 5.5, 5.6
Today's topic: Rolle's Theorem

Any question from previous class?

Fractional exponents

Let $g(x) = x^{2/3}(x-1)^3$.

Find local and global extrema of g on $[-1, 2]$.

Warm-up

State Rolle's Theorem.

Trivia: Who was Rolle?

How many zeroes?

Let

$$f(x) = e^x - \sin x + x^2 + 10x$$

How many zeroes does f have?

Zeroes of the derivative

Sketch the graph of a function f that is differentiable on $\mathbb{R}$ and such that

1. f has exactly 3 zeroes and f' has exactly 2 zeroes.
2. f has exactly 3 zeroes and f' has exactly 3 zeroes.
3. f has exactly 3 zeroes and f' has exactly 1 zero.
4. f has exactly 3 zeroes and f' has infinitely many zeroes.

Zeroes of a polynomial

You probably learned in high school that a polynomial of degree n has at most n real zeroes. Now you can prove it!

Hint: Use induction. If you are having trouble, try the case $n = 3$ first.

The second Theorem of Rolle

Complete statement for this theorem and prove it.

Rolle's Theorem 2

Let $a < b$. Let f be a function defined on $[a, b]$.
IF

- (Some conditions on continuity and derivatives)
- $f(a) = f'(a) = 0$
- $f(b) = 0$

THEN $\exists c \in (a, b)$ such that $f''(c) = 0$.

Hint: Apply the 1st Rolle's Theorem to f, then do something else.

A new theorem

We want to prove this theorem:

Theorem 1

Let f be a differentiable function on an open interval I.
IF $\forall x \in I, f'(x) \neq 0$
THEN f is one-to-one on I.

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1. Transform $[P \implies Q]$ into $[(\text{not } Q) \implies (\text{not } P)]$.
   You get an equivalent Theorem (call it "Theorem 2").
   We are going to prove Theorem 2 instead.
2. Write the definition of "f is not one-to-one on I". You will need it.
3. Recall the statement of Rolle's Theorem. You will need it.
4. Do some rough work if needed.
5. Write a complete proof for Theorem 2.