

## Welcome back to MAT137- Section L5101

- Welcome back after the Reading Week!
- Assignment #4 due on Nov 26.
- Test 2 opens on Dec 4
- Assignment #5 due on Dec 20.
- **Next class: Local Extrema**
  - **Watch videos 5.2, 5.3, 5.4 (and 5.1)**

Let's get started!!

Today's videos: 4.12-4.14

Today's topic: Inverse trigonometric functions

Any question from previous class?

## Definition of arctan

1. Sketch the graph of  $\tan$ .
2. Prove that  $\tan$  is not one-to-one.
3. Select the largest interval containing 0 such that the restriction of  $\tan$  to it is one-to-one. We define  $\arctan$  as the inverse of this restriction. Let  $x, y \in \mathbb{R}$

$$\arctan y = x \iff ???$$

4. What is the domain of  $\arctan$ ? What is the range of  $\arctan$ ? Sketch the graph of  $\arctan$ .
5. Compute
  - 5.1  $\arctan(\tan(1))$
  - 5.2  $\arctan(\tan(3))$
  - 5.3  $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$
  - 5.4  $\arctan(\tan(-6))$
  - 5.5  $\tan(\arctan(0))$
  - 5.6  $\tan(\arctan(10))$

## Derivative of arctan

Obtain (and prove) a formula for the derivative of arctan.

*Hint:* Call  $f(t) = \arctan t$  and differentiate

$$\forall t \in \dots \quad \tan(f(t)) = t$$

## Trig-inverse-trig

Find simple expressions for these quantities and state the domain on which they are valid:

1.  $\sin(\arccos x)$

2.  $\sec(\arccos x)$

3.  $\sec(\arctan x)$

4.  $\tan(\operatorname{arcsec} x)$

## Trig-inverse-trig

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1.  $\sin(\arccos x)$

3.  $\sec(\arctan x)$

2.  $\sec(\arccos x)$

4.  $\tan(\operatorname{arcsec} x)$

*Hint:* There are two standard ways to attack these problems:

- Use a trig identity  
e.g.: a trig identity relating  $\sin$  and  $\cos$  for (1)
- Or draw a right triangle with side lengths 1 and  $x$   
e.g.: with an angle  $\theta$  such that  $\cos \theta = x$  for (1)

If you need to take a square root, you must justify which branch (+ or -) you are choosing.

1. Complete: “We define arcsec as the inverse of the restriction of sec to ...”  
*Hint:* Sketch the graph of sec.
2. What are the domain and range of arcsec?  
Sketch its graph.
3. Obtain (and prove) a formula for the derivative of arcsec in the same way you did for arctan.
4. Now obtain the same formula in a different way: use  $\sec x = \frac{1}{\cos x}$  to write arcsec in terms of arccos.