

## Welcome back to MAT137- Section L5101

- Assignment 7 due tomorrow.
- Assignment 8 due on March 4.
- Test 4 opens on March 12.
  
- Thursday: Watch videos 11.7 and 11.8
- Questions from previous class?

Let's get started!!

Today's video: 11.5 and 11.6 !!

Today's topic: Theorems about sequences!

## True or False - Rapid fire

1. (convergent)  $\implies$  (bounded)
2. (convergent)  $\implies$  (monotonic)
3. (convergent)  $\implies$  (eventually monotonic)
4. (bounded)  $\implies$  (convergent)
5. (monotonic)  $\implies$  (convergent)
6. (bounded + monotonic)  $\implies$  (convergent)
7. (divergent to  $\infty$ )  $\implies$  (eventually monotonic)
8. (divergent to  $\infty$ )  $\implies$  (unbounded above)
9. (unbounded above)  $\implies$  (divergent to  $\infty$ )

## Fill in the blanks

Let  $\{a_n\}$  be a decreasing, bounded sequence.

Assume  $a_1 = 1$  and  $a_n$  is never 0.

Let  $m$  be the greatest lower bound of  $\{a_n\}$ .

For each of the statements below, find **all** the values of  $m$  that make the statement true.

1. IF  THEN  $\{1/a_n\}$  is bounded
2. IF  THEN  $\{1/a_n\}$  is increasing
3. IF  THEN  $\{\sin a_n\}$  is bounded
4. IF  THEN  $\{\sin a_n\}$  is decreasing

## Proof of Theorem 3

Write a proof for the following Theorem

### Theorem 3

Let  $\{a_n\}_{n=0}^{\infty}$  be a sequence.

- IF  $\{a_n\}_{n=0}^{\infty}$  is increasing AND unbounded above,
- THEN  $\{a_n\}_{n=0}^{\infty}$  is divergent to  $\infty$

## Proof of Theorem 3

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### Theorem 3

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1. Write the definitions of “increasing”, “unbounded above”, and “divergent to  $\infty$ ”
2. Using the definition of what you want to prove, write down the structure of the formal proof.
3. Do some rough work if necessary.
4. Write a formal proof.

## Proof feedback

1. Does your proof have the correct structure?
2. Are all your variables fixed (not quantified)? In the right order? Do you know what depends on what?
3. Is the proof self-contained? Or do I need to read the rough work to understand it?
4. Does each statement follow logically from previous statements?
5. Did you explain what you were doing? Would your reader be able to follow your thought process without reading your mind?

## Critique this proof - #1

- $\forall M \in \mathbb{R}, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n_0 \implies x_n > M$
- $M$  is not an upper bound:  $\exists n_0 \in \mathbb{N}$  s.t.  $x_{n_0} > M$
- $n \geq n_0 \implies x_n \geq x_{n_0} > M$

## Critique this proof - #2

- WTS  $a_n \rightarrow \infty$ . This means:  
 $\forall M \in \mathbb{R}, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n_0 \implies x_n > M$
- bounded above:  $\exists M \in \mathbb{R}, \forall n \in \mathbb{N}, x_n \leq M$
- negation:  $\forall M \in \mathbb{R}, \exists n \in \mathbb{N}, x_n > M$
- $\forall n \in \mathbb{N}$ , take  $n = n_0$ .