1. Course Outline

In these notes, we develop some basic techniques in solving partial differential equations and analyzing their solutions. The long term goal is to understand principal evolution equations. We establish an existence theory for the selected class of equations, describe their key properties, isolate their most important solutions and study stability or instability of these solutions. Some of non-evolution equations appear as static equations for the evolution ones.

Given the time constrain, we have to be very selective about the equations we consider. The guiding principles are the importance of the equations in mathematics and in applications, relevance to the current research and the central role of the techniques needed to analyze these equations.

2. Texts

L. C. Evans, Partial Differential Equations, AMS
R. McOwen, Partial Differential Equations, Prentice Hall, 2003;

3. Marking scheme

Breakup of the grade: Class participation/tw presentations 30%/70%.

4. Presentations

The presentations, 20 min, will take places on Thursdays, during the class hours.

Each presentation should contain a nontrivial mathematical ingredient. Independently of how the papers or book chapters whose material you are presenting are structured, your presentations must follow the following format:

Statement of the problem
Statement of the result
Outline of the proof/derivation.

The introductions should be no more than 1-2 sentences. You should present the problems and results in the simplest non-trivial forms without discussions of the generalizations, applications, etc. The outlines of the proofs should contain the main, key ideas which contain an interesting mathematics. You can use transparencies (computer or otherwise) for long equations, graphs and some biological background. The mathematics (i.e. definitions, results, ideas of the proof) should be done on the blackboard.

5. Schedule of Presentations

- January 16: 1) Fourier transform and its applications to partial differential equation (Appendices A.1 and A.2 of instructor’s notes) (EM)
- January 23:
  2) Sobolev imbedding theorems ([28] Section 6.4a, b, d, or [16], Section 5.6) (JD) and
  3) Kondrashov compactness theorem ([28] Section 6.4c, or [16], Section 5.7) (EJ)
- January 30:
  4) Functionals on real and complex spaces (Section 5.1, 5.2, 5.6 of the instructor’s notes) (TK)
  5) Gâteaux and Fréchet derivatives (Section 4 of [31], see also [28] Section 7.1, 7.3) (DC)
- February 6:
  6) Nonlinear eigenvalue problem (Section 6.2 of the instructor’s notes) (AT) and
  7) Minimization problem with constraints, Lagrange multipliers and spectrum (Subsection 5.7 of the instructor’s notes) (AZ)

Date: January 17, 2015.
February 13:
8) Operators (Appendix B.1 - B.2 of the instructor’s notes) (JHB)
9) Spectra of operators and Perron-Frobenius theory (Appendix B.3 - B.4 of the instructor’s notes) (SP)

March 6:
10) Semigroups (Section C of the instructor’s notes) (MS)
11) Existence of the ground state for nonlinear Schrödinger equations and the Nirenberg - Gagliardo inequality ([37], Introduction and Section II, see also [4], Sections 1 (in particular, Theorem 1 i) - ii) and 3 and Appendix, here take a special case of \( g(s) = s^\sigma \), with appropriate \( \sigma \) (JD)

March 13:
12) The implicit function theorem (Subsection 5.1 of [31]) (ST)
13) & 14) Asymptotics of the heat equations ([2]). (AZ & TK)
15) Stability of cylinders under the volume preserving mean curvature flow ([24]) (AT)

March 20:
16) Local existence for the Navier-Stokes equation ([28] Subsection 11.4a) (EM)
17) Remainder terms in Sobolev inequalities ([5], see also [19]) (EJ)
18) Hamilton equations (Section 8.2 of the instructor’s notes) (IBN)

March 27:
19) Schauder fixed point theorem ([28] Subsection 9.1) (DC)
20) Semigroups and diffusion processes ([30], Section 8, [34], Sections 9.1-9.5) (MS)
21) Stability of translating solitons under the mean curvature flow ([14]) (IBN)
22) Control theory and the Hamilton-Jacobi-Bellman equation (Sections 10.3.1 - 10.3.3 of [16]) (JHB)
23) Variational problems on vector bundles ([18], Section 2) (ST)

25) Existence of magnetic vortices ([3])
27) The Abrikosov lattice solutions of the Ginzburg-Landau equations (Section 10 of [31], see also [35, 21])
28) The Keller - Segel equations of chemotaxis ([11]).

**REFERENCES**