Breaking The Rules

1. What are the Rules?

In order to break the rules, we must first know what they are.

(1) In your group, come up with some rules in math that you learned in class. For example, what happens when you double a number? What happens when you triple a number? Is there a largest number? What happens when you multiply a number by 3? Can you multiply two numbers to get 0? Can you add two positive numbers and get 0? What other rules can you come up with?

(2) Following these rules, what happens when you do $1 + 1 + 1 + \cdots + 1$ a certain number of times? What happens when you keep doing it?

(3) Following these rules, what happens when you do $2 \times 2 \times 2 \times \cdots \times 2$ a certain number of times? What happens when you keep doing it?

(4) In the above, what if I replace 2 with 10? What about replacing it with your favourite number?

(5) What do you know about remainders? Give an example of a remainder.

(6) What is the remainder when we divide a negative number by a positive number? For example, what is the remainder of $(-11) \div 5$?

2. Let's Make Crazy Rules

Let's start off by making a new addition. Replace the sign "+" with a new sign "$\oplus$" with a new addition law. For example, can you spot the addition law in the following pattern:

$$1 \oplus 1 = 3 \qquad 0 \oplus 0 = ?$$
$$2 \oplus 1 = 4 \qquad 6 \oplus 8 = ?$$
$$1 \oplus 2 = 4 \qquad (-1) \oplus 3 = ?$$
$$2 \oplus 2 = 5 \qquad (-5) \oplus 5 = ?$$

What about the following pattern:

$$1 \oplus 1 = 1 \qquad 0 \oplus 0 = ?$$
$$1 \oplus 2 = 1 \qquad 6 \oplus 8 = ?$$
$$2 \oplus 1 = 2 \qquad (-1) \oplus 2 = ?$$
$$1 \oplus 4 = 1 \qquad 8 \oplus 6 = ?$$
$$3 \oplus 5 = 3 \qquad 0 \oplus (-100) = ?$$

Spend some time coming up with your own addition law, and see how weird it can get!

3. A Special Kind of Rule-Breaking

Here is a special kind of broken addition. See if you can complete the pattern:

$$1 \oplus 1 = 2 \qquad (-4) \oplus 0 = 1$$
$$1 \oplus 3 = 4 \qquad (-2) \oplus (-3) = 0$$
$$1 \oplus 4 = 0 \qquad 3 \oplus 3 = ?$$
$$2 \oplus 4 = 1 \qquad 4 \oplus 3 = ?$$
$$2 \oplus 3 = 0 \qquad 4 \oplus 4 = ?$$
$$0 \oplus 6 = 1 \qquad 8 \oplus 9 = ?$$
$$7 \oplus 9 = 0 \qquad (-1) \oplus (-8) = ?$$

For the above addition law:

(1) What happens when you do $1 \oplus 1 \oplus 1 \oplus \cdots \oplus 1$ a certain number of a times? What happens when you keep doing it?

(2) Complete the following table:

$\oplus$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Do we need to include numbers larger than 4?

(3) What patterns do you notice in the table above?

(4) Can you try to come up with a multiplication law that operates in the same way as the second addition law? Call this new multiplication law $\otimes$. Under this multiplication law, what is $3 \otimes 4$? What is $2 \otimes 4$?
(5) Can you find a number x such that $x \otimes 4 = 1$? It might help to complete the following table:

$\otimes$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

(6) What happens when we look at $3, 3 \otimes 3, 3 \otimes 3 \otimes 3$, and so on...

4. Laws and Symmetry

Let's go back to the funny addition law, $\oplus$.

(1) Draw a 5-sided star on this page. What kind of symmetry does it have?
(2) Can you rotate the page so that the star looks the same?
(3) How many times do you need to rotate the page to make it face you again?
(4) How does this relate to $1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 = 0$?