

Welcome to MAT135 LEC0101 (Assaf)



As you walk in, look up the lyrics to your favourite song. Are there any math-y words in the lyrics?



S3.3 – We are all **products** of our environments

Assaf Bar-Natan

“ Can't seem to get no rest?
Put our product to the test
You'll feel just fine ”

–“ The Big Bright Green Pleasure Machine ”, Simon and Garfunkel

Oct. 18, 2019

Interpreting Leibniz

In Nov. 11, 1675, Leibniz wrote the following in his notebook (translated from Latin):

Let us now examine whether $dx dy$ is the same thing as \overline{dxy} , and whether dx/dy is the same thing as $d\frac{x}{y}$; it may be seen that if $y = z^2 + bz$, and $x = cz + d$; then

$$dy = 2z + 2\beta z + \beta^2, + bz + b\beta, - z^2 - bz,$$

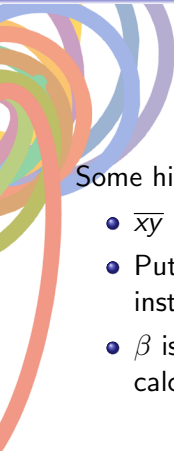
and this becomes $dy = 2z + b\beta$.

In the same way $dx = c\beta$, and hence

$$dx dy = \overline{2z + b} c\beta^2.$$

But you get the same thing if you work out \overline{dxy} in a straightforward manner. For in each of the several factors there is a separate destruction, the one not influencing the other; and it is the same thing in the case of divisors.

Desciphering Leibniz



Some hints:

- \overline{xy} is notation for (xy)
- Putting a d in front of a quantity means “find the instantaneous rate of change”
- β is an *infinitesimal* quantity, a concept that was used in early calculus. Feel free to ignore it.

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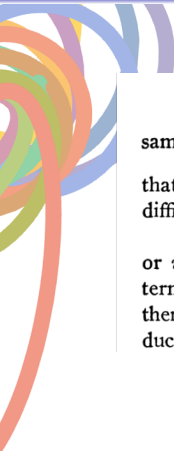
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Leibniz' Example



Hence it appears that it is incorrect to say that $dv d\psi$ is the same thing as $dv\psi$, or that $\frac{dz}{d\psi} = d\frac{z}{\psi}$; although just above I stated that this was the case, and it appeared to be proved. This is a difficult point. But now I see how this is to be settled.

If we have v and ψ , and they form some quantity, say $\phi = v\psi$ or v/ψ , and if the values of v and ψ are expressed as rationals in terms of some one thing, for instance, in terms of the abscissa x , then the calculus will always show that the same difference is produced, and that $d\phi$ is the same as $dv d\psi$ or $dv/d\psi$. But now I see

the former can never happen, nor can it come to the latter by separation of parts; for example,

$$x + \beta, \cap x + \beta, -, x, x, \text{ becomes } 2\beta x,$$

which is quite a different thing from

$$x + \beta, -x, \cap x + \beta, -x \text{ which gives } \beta^2.$$

Hence it must be concluded that $dv\psi$ is not the same as $dv d\psi$, and

Leibniz' Correction

MANUSCRIPT DATED NOV. 21, 1675.

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Let us seek to obtain others in addition, such as

$$\int t \, dy = \int y \, dx.$$

Now this furnishes us with nothing new; but $\int tw + \int xw = xy$

or $t \, dy + x \, dy = \overline{dxy}$, and $t = \frac{dx}{dy} y$; hence the latter = $\frac{\overline{dxy} - x \, dy}{dy}$.

Therefore $\overline{dx} y = \overline{dxy} - x \, dy$.

Now this is a really noteworthy theorem and a general one for all curves. But nothing new can be deduced from it, because we had already obtained it.

However, from another principle we shall obtain a new theorem; for it is known that the sum of every BP = $BC^2/2$; that is to

say, BP = $\frac{a^2}{t-x}$, $t = \frac{\beta y}{w} = \frac{\overline{dx}}{dy} y$, and therefore

$$BP = \frac{a^2 dy}{dx y - \overline{dy} x} = \frac{\overline{dy^2}}{2} \dots \dots \dots (3)$$

We therefore have two equations, in which dy occurs, namely:

Quotient Rule and Constant Rule

The quotient rule tells us:

$$\frac{d}{dx} \left(\frac{f(x)}{c} \right) = \frac{c \frac{d}{dx} f(x) - f(x) \frac{d}{dx} c}{c^2}$$

and the constant multiple rule tells us:

$$\frac{d}{dx} \left(\frac{f(x)}{c} \right) = \frac{1}{c} \frac{d}{dx} f(x)$$

Do these rules agree? Justify your response in a short paragraph. Be sure to include formulas!

Plans for the Future



For Monday:
WeBWork 3.4 and read section 3.4