## EXERCISE: THE COUPLED PENDULUM

The equation of motion for two coupled pendula with same mass $m$ and length $l$ reads

$$mx'' = -\alpha x - k(x - y)$$
$$my'' = -\alpha y - k(y - x)$$

with $\alpha = mg/l$. (Here, $g$ denotes the earth acceleration.) The origin of the coordinate system is chosen such that $x = y = 0$ corresponds to the equilibrium.

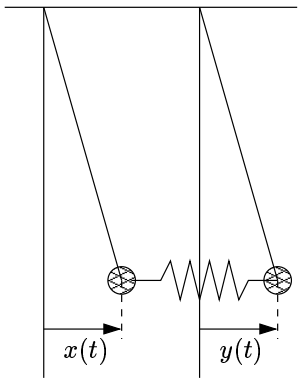


FIGURE 1. Coupled pendula

Find the general solution of the system. Note that this is a system of two second-order ODEs. So it is equivalent to a system of *four* first order ODEs. You have to find 4 linearly independent solutions of this system! You can either do this by using a generalization of what we have done in class for $2 \times 2$-matrices to $4 \times 4$-matrices or you can use some physical intuition: 2 solutions can be found by assuming the pendula move parallel, and 2 other solutions can be found by assuming the pendula have the same period but move in opposite directions. (In both cases the system reduces to a second order ODE of one variable which you can solve).

Discuss, what happens if at the time $t = 0$ you push one of the pendula, i.e. discuss the solution with initial condition $x(0) = y(0) = y'(0) = 0$, $x'(0) = 1$.